NOTATION

x, y, z	are the Cartesian coordinates;
T, v	are the perturbations of temperature and z component of velocity;
Φ	is the potential of magnetic field perturbations;
M	is the magnetization of liquid;
H	is the magnetic field intensity;
μ_0	is the magnetic permeability of vacuum;
χ	is the magnetic susceptibility;
η	is the dynamic viscosity;
λ	is the thermal conductivity;
$\varkappa$	is the thermal diffusivity;
g	is the acceleration due to gravity;
β	is the volume coefficient of expansion;
K	is the pyromagnetic coefficient;
l	is the layer thickness;
γ	is the temperature gradient;
$\mathbf{k} = [k_x, k_y, 0]$	is the wave vector;
α	is the surface tension;
$\sigma = -(\partial\alpha/\partial T)/$	
α^*; μ	is the relative magnetic permeability;
F	is the amplitude of perturbations of free surface.

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CONVECTIVE MOTION OF A CONDUCTING LIQUID IN AN ELECTROMAGNETIC FIELD, TAKING INTO ACCOUNT FINITE WALL THICKNESS AND THERMAL CONDUCTIVITY

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The effect of the temperature-dependent electrical conductivity of the liquid and the finite wall thickness and the thermal conductivity on stability is investigated in a linear formulation.

In [1] the convective instability of a liquid layer in a magnetic field was investigated, taking into account the finite wall thickness and thermal conductivity. In the present work, stability of this type is investigated taking account of the temperature dependence of the electrical conductivity.

1. Formulation of the Problem

Consider an infinite horizontal layer of electrically conducting liquid of thickness B, the electrical conductivity of which depends linearly on the temperature $\sigma=\sigma_{00}\,[1+\alpha(T-T_{00})]$ under the condition that $|\alpha(T-T_{00})|\ll 1$ [2]. The walls bounding the layer have the same finite thickness and thermal conductivity λ_1. The temperatures at the external surfaces of the walls are given to be constant, but different (T_1 is the temperature at the lower wall and T_2 at the upper wall). In the y direction, a constant external electric field

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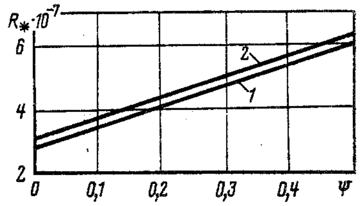

Fig. 1. Critical number R_* as a function of ψ with a_1 = 0.1 for different values of H: 1) H = 0.5; 2) 8.5.

of strength E_0 is applied, and in the x direction a constant magnetic field of induction B. The magnetic field induced by the currents flowing through the liquid are assumed small in comparison with the external field under the condition

$$\frac{\mu_0 \sigma_{00} E_0 b}{B} \ll 1, \quad \mathrm{Re}_m \ll 1,$$

where μ_0 is the magnetic permeability; Re_m is the magnetic Reynolds number.

The equilibrium state is characterized by the following distribution of quantities marked by the subscript 0:

$$\mathbf{v}_0 = 0, \quad \mathbf{E}_0 = \mathbf{E}_0(0,\ E_0,\ 0), \quad \mathbf{B}_0 = \mathbf{B}_0(B,\ 0,\ 0),$$

$$p_0 = \sigma_{00} E_0 Bz + \text{const}, \quad T_0 = -Sz^2 + \frac{S(1+2\psi)-1}{2\psi+1}(z+\psi)+1. \tag{1}$$

Here $S = \sigma_{00} E_0^2 b^2 / 2\lambda\Delta T$ characterizes the current strength; $\Delta T = T_1 - T_2$; $\psi = \lambda b_1/\lambda_1 b$ characterizes the heat conduction of the liquid and the wall [1]; λ is the thermal conductivity of the liquid.

In a linear formulation, the initial system of equations for the perturbation takes the form [1-4]

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \Delta u, \tag{2}$$

$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu \Delta v - \sigma_{00} B \left(\frac{\partial \varphi}{\partial z} + vB\right), \tag{3}$$

$$\rho \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} + \mu \Delta w + \sigma_{00} B \left(\frac{\partial \varphi}{\partial y} - wB\right) - E_0 B \sigma + \rho g \beta T, \tag{4}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{5}$$

$$\sigma_{00} \Delta \varphi - \sigma_{00} B \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) - E_0 \frac{\partial \sigma}{\partial y} = 0, \tag{6}$$

$$\rho c_p \left(\frac{\partial T}{\partial t} + \frac{dT_0}{dz} w\right) = -2\sigma_{00} E_0 \left(\frac{\partial \varphi}{\partial y} - wB\right) + E_0^2 \sigma + \lambda \Delta T, \tag{7}$$

$$\sigma = \sigma_{00} \alpha T. \tag{8}$$

All the notation here is as commonly used.

2. Perturbation Equation for Vertical Velocity-Vector Component. Perturbation-Monotonicity Condition

Differentiating Eqs. (2), (3), and (4) with respect to x, y, and z, respectively, adding them, and using Eq. (5), the following relation is obtained:

$$-\Delta p-\sigma_{00}B^2\left(\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)-E_0B\frac{\partial\sigma}{\partial z}+\rho g\beta\frac{\partial T}{\partial z}=0. \tag{9}$$

It follows from Eqs. (4) and (9) that

$$\rho\frac{\partial\Delta w}{\partial t}=\mu\Delta^2w+\sigma_{00}B^2\frac{\partial^2v}{\partial y\partial z}-E_0B\Delta_1\sigma+\sigma_{00}B\frac{\partial\Delta\varphi}{\partial y}-\sigma_{00}B^2\Delta_1w+g\rho\beta\Delta_1T,$$
$$\Delta_1=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}. \tag{10}$$

Using Eq. (6), the following result is obtained from Eq. (10):

$$\rho\frac{\partial\Delta w}{\partial t}=\mu\Delta^2w-E_0B\frac{\partial^2\sigma}{\partial x^2}-\sigma_{00}B^2\frac{\partial^2w}{\partial x^2}+g\rho\beta\Delta_1T. \tag{11}$$

Applying the operator $(\partial/\partial t-\varkappa\Delta-K)$ to both sides of Eq. (11), and using Eqs. (7) and (8), the following equation is obtained:

$$\left(\frac{\partial}{\partial t}-\varkappa\Delta-K\right)\left(\frac{\partial}{\partial t}-\nu\Delta\right)\Delta w=-\frac{\sigma_{00}B^2}{\rho}\left(\frac{\partial}{\partial t}-\varkappa\Delta-K\right)\times$$
$$\times\frac{\partial^2w}{\partial x^2}+\frac{2\sigma_{00}^2E_0^2B\alpha}{\rho^2c_p}\frac{\partial^3\varphi}{\partial^2x\partial y}-\frac{E_0B\sigma_{00}\alpha}{\rho}\left(\frac{B\sigma_{00}E_0}{\rho c_p}-\frac{dT_0}{dz}\right)\times$$
$$\times\frac{\partial^2w}{\partial x^2}-g\beta\frac{dT_0}{dz}\Delta_1w-\frac{2\sigma_{00}E_0}{\rho c_p}g\beta\frac{\partial\Delta_1\varphi}{\partial y}+\frac{2\sigma_{00}E_0\beta g\Delta_1w}{\rho c_p}, \tag{12}$$

where $\varkappa=\dfrac{\lambda}{\rho c_p}$; $K=\dfrac{E_0^2\sigma_{00}\alpha}{\rho c_p}$, $\nu=\dfrac{\mu}{\rho}$.

The solution of Eq. (12) is sought in the form [1, 2]

$$w=W(z)\cos(a_1x/b)\cos(a_2y/b)\exp nt,$$
$$T=\theta(z)\cos(a_1x/b)\cos(a_2y/b)\exp nt,$$
$$\varphi=\eta(z)\cos(a_1x/b)\sin(a_2y/b)\exp nt.$$

Then a single equation for the amplitude and the perturbation is obtained:

$$[MP^{1/2}-(D^2-a^2)-A][MP^{-1/2}-(D^2-a^2)](D^2-a^2)W=$$
$$=H^2[MP^{1/2}-(D^2-a^2)-A]a_1^2W-2AH^2a_1^2W-2AH^2\frac{a_1^2a_2}{Bb}\eta+$$
$$+\frac{2gb^3E_0^2\sigma_{00}\beta a^2a_2}{\lambda\nu}\eta-[L(1/2-\zeta)+\Phi R]a_1^2W-R_2a^2W+[R_3(1/2-\zeta)+\Phi R_1]a^2W, \tag{13}$$

where

$$D=\frac{d}{d\zeta};\quad \zeta=\frac{z}{b};\quad a^2=a_1^2+a_2^2;\quad P=\frac{\nu}{\varkappa};$$
$$M=\frac{nb^2}{(\varkappa\nu)^{1/2}};\quad H^2=\frac{\sigma_{00}B^2b^2}{\mu};\quad A=\frac{\sigma_{00}E_0^2\alpha B^2}{\lambda};$$
$$R=\frac{E_0B\sigma_{00}\alpha b^3\Delta T}{\varkappa\mu};\quad R_1=\frac{g\beta b^3\Delta T}{\varkappa\nu};\quad \Phi=-\frac{1}{2\psi+1};$$
$$R_2=\frac{2\sigma_{00}E_0B\beta gb^4}{\lambda\nu};\quad R_3=\frac{g\beta b^5\sigma_{00}E_0^2}{\varkappa\nu\lambda};\quad L=\frac{\sigma_{00}^2E_0^3B\alpha b^5}{\lambda\mu\varkappa}.$$

If the influence of Joule heating may be neglected, the terms containing A, L, and R_3 drop out of the right-hand side of Eq. (13). Then, following [2], it may be shown that in the case of heating from below ($T_1>T_2$), the principle of monotonic change in stability is satisfied provided that

$$E_0B\sigma_{00}\alpha<\rho g\beta. \tag{14}$$

In contrast to the case where the temperature dependence of the electrical conductivity is neglected [1], the fact that $\alpha = 0$ means that Eq. (14) is always satisfied. At sufficiently large electrical field strength and magnetic induction, there may then be oscillations in the liquid during heating from below, whereas during heating from above the oscillations are quenched if Eq. (14) is satisfied (in the particular case when $B = 0$ or $E_0 = 0$).

In the general case, it is assumed that the principle of monotonic change in stability is satisfied. If the condition $|\alpha(T - T_{00})| \ll 1$ is to be satisfied, then it is necessary to have $|A| \ll 1$ [3], when all the terms containing A may be neglected in Eq. (13), and it may be assumed that $M = 0$. The result obtained is

$$(D^2 - a^2)[(D^2 - a^2)^2 + H^2 a_1^2] W = - a_1^2 RQW, \tag{15}$$

where

$$Q \equiv \left\{R_4(1/2 - \zeta) + \Phi - R_5 \frac{a^2}{a_1^2}(1/2 - \zeta) - R_6 \frac{a^2}{a_1^2}\Phi + R_7 \frac{a^2}{a_1^2}\right\};$$

$$R_4 = \frac{\sigma_{00} E_0^2 b^2}{\lambda \Delta T}; \quad R_5 = \frac{g\beta b^2 E_0 \rho}{\lambda B \alpha \Delta T}; \quad R_6 = \frac{g\beta\rho}{E_0 B \alpha \sigma_{00}}; \quad R_7 = \frac{2bg\beta}{\alpha c_p \Delta T}.$$

The boundary conditions for solid surfaces take the form

$$W = DW = [(D^2 - a^2)^2 + H^2 a_1^2] W = 0 \quad \text{when } \zeta = 0, 1. \tag{16}$$

Note that setting $\psi = 0$ and $\beta = 0$ in Eq. (15) gives the case studied in [3], and $\alpha = 0$ gives the case studied in [1].

3. Bubnov – Galerkin Method

A version of the Bubnov–Galerkin method is used to solve Eq. (15) with the boundary conditions (16) [4, 5]. It is expedient to replace Eq. (15) by the system

$$F = [(D^2 - a^2)^2 + Ha_1^2] W, \tag{17}$$

$$(D^2 - a^2) F = - a_1^2 RQW. \tag{18}$$

The solution of Eq. (17) is sought in the form

$$F = \sum_{m=1}^{\infty} E_m \sin m\pi\zeta, \tag{19}$$

where E_m are constant. It is obvious that Eq. (19) satisfies the condition $F = 0$ when $\zeta = 0, 1$.

Next, W is written in the form of a series:

$$W = \sum_{m=1}^{\infty} E_m W_m. \tag{20}$$

Substituting Eqs. (19) and (20) into Eq. (17), an equation for determining W_m is obtained:

$$[(D^2 - a^2)^2 + H^2 a_1^2] W_m = \sin m\pi\zeta \tag{21}$$

with the boundary conditions

$$W_m = DW_m = 0 \quad \text{when } \zeta = 0, 1. \tag{22}$$

The general solution of Eq. (21) takes the form

$$W_m = \gamma_m^{-1}(A_m \cos\lambda_2\zeta \operatorname{ch}\lambda_1\zeta + B_m \cos\lambda_2\zeta \operatorname{sh}\lambda_1\zeta + C_m \sin\lambda_2\zeta \operatorname{ch}\lambda_1\zeta + D_m \sin\lambda_2\zeta \operatorname{sh}\lambda_1\zeta + \sin m\pi\zeta), \tag{23}$$

where

$$\gamma_m = (m^2\pi^2 + a^2)^2 + H^2 a_1^2; \quad \lambda_{1,2} = \{1/2[(a^4 + a_1^2 H^2)^{1/2} \pm a^2]\}^{1/2}.$$

The constants A_m, B_m, C_m, and D_m are determined from the boundary conditions (22):

$$A_m = 0, \quad B_m = -\lambda_1^{-1}(m\pi + \lambda_2 C_m),$$

$$C_m = [m\pi\{(-1)^{m+1}\lambda_1 - \lambda_2 \sin\lambda_2 \operatorname{sh}\lambda_1 + \lambda_1 \cos\lambda_2 \operatorname{ch}\lambda_1 -$$

$$-(\lambda_2 \cos\lambda_2 \operatorname{sh}\lambda_1 + \lambda_1 \sin\lambda_2 \operatorname{ch}\lambda_1 | \operatorname{ctg}\lambda_2)]/[(\lambda_1^2+\lambda_2^2)\sin\lambda_2 \operatorname{sh}\lambda_1 -$$
$$-(\lambda_2 \cos\lambda_2 \operatorname{sh}\lambda_1 + \lambda_1 \sin\lambda_2 \operatorname{ch}\lambda_1)(\lambda_1 \operatorname{cth}\lambda_1 - \lambda_2 \operatorname{ctg}\lambda_2)],$$
$$D_m = \lambda_1^{-1}[m\pi \operatorname{ctg}\lambda_2 - C_m(\lambda_1 \operatorname{cth}\lambda_1 - \lambda_2 \operatorname{ctg}\lambda_2)].$$

Substituting Eqs. (19) and (20) into Eq. (18) yields

$$\sum_{m=1}^{\infty} E_m [(m^2\pi^2 + a^2)\sin m\pi\zeta - a_1^2 RQ] W_m = 0. \tag{24}$$

Multiplying Eq. (24) by $\sin l\pi\zeta$ and integrating with respect to ζ from 0 to 1, a system of homogeneous linear equations for E_m is obtained:

$$\sum_{m=1}^{\infty} E_m \left\{\frac{\gamma_m (m^2\pi^2 + a^2)}{2a_1^2 R}\delta_{ml} - \left(\frac{m}{l}\right)\right\} = 0.$$

For a nontrivial solution, the determinant of the coefficients must be zero:

$$\left\| \frac{\gamma_m (m^2\pi^2 + a^2)}{2a_1^2 R}\delta_{ml} - \left(\frac{m}{l}\right) \right\| = 0, \tag{25}$$

where

$$\left(\frac{m}{l}\right) = \int_0^1 Q\gamma_m W_m \sin l\pi\zeta d\zeta.$$

Equation (25) gives the critical number R*, which characterizes the temperature dependence of the electrical conductivity [2], as a function of the parameters of the problem. For fixed R_4, R_5, R_6, R_7, and Φ, using a fourth-order approximation (l = 4), the critical number R_* is found, as well as the corresponding minima of the wave numbers a_* (for fixed a_1) and a_{1*} (for fixed a_*).

Calculations on an ES-1022 computer show that R_* decreases with increase in a_1. In the case of no Joule heating, as expected, the critical number R_* increases with rise in ψ and H (Fig. 1).

NOTATION

u, v, w	are the velocity components along the x, y, and z axes;
T, p,	are the temperature and pressure;
λ, σ, ν, β	are the thermal and electrical conductivity, kinematic viscosity, volume-expansion coefficient;
ρ	is the density;
g	is the acceleration due to gravity;
α	is the coefficient taking into account change in conductivity of the liquid with temperature;
φ	is the perturbation of electric field strength;
$\mu = \rho\nu$	

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